J. Fluid Mech. (1972), *vol.* 52, *part* 3, *pp.* 593–608
Printed in Great Britain

The collection efficiencies of small droplets falling through a sheared air flow

By P. R. JONAS AND P. GOLDSMITH

Cloud Physics Laboratory, Meteorological Office, Bracknell

(Received 30 September 1971)

Experiments in which collection efficiencies have been measured for small droplets falling through a sheared air flow are described. The results indicate that these collection efficiencies are much greater than for the same droplets falling in still air when the collecting droplets are less than about 25 μm in radius, but that for larger droplets there is very little effect. Calculations of the collision efficiencies of rigid spheres moving in a sheared air flow are presented but these fail to account for the experimental results. These discrepancies are discussed.

1. Introduction

Cloud droplets can grow both by condensation and by colliding and coalescing with larger droplets. The rate of growth by condensation in conditions typical of a cloud has been shown to be small when the droplet radii exceed about 15 μm (Mason 1971). Calculations of the rate of growth of cloud droplets by collision and coalescence have been made by Bartlett (1970) and others, assuming that the droplets are falling in still air. The results indicate that the rate of growth is small unless the cloud contains an appreciable number of droplets of radii greater than about 20 μm. On the basis of the calculations the observed rate of production of drops of radii greater than about 40 μm cannot be explained. The effects of the electric fields or droplet charges existing in clouds during the early stages of growth have been shown both experimentally (Woods 1965) and theoretically (Sartor 1957) to be unimportant. The effect of turbulence in clouds has not been so extensively studied but Bartlett & Jonas (1972) have shown that the turbulence cannot appreciably affect the growth rate by condensation.

The growth of droplets by collision and coalescence depends on the collection efficiency of the droplets. This can be defined for a pair of droplets of radii R and r as $A/\pi(R+r)^2$, where A is the cross-sectional area of the volume within which the centre of the smaller droplet must lie if it is to coalesce with the larger droplet. Turbulence could affect the collection efficiencies of droplets in a cloud and thereby alter the rate of production of larger droplets. Woods, Drake & Goldsmith (1972) have argued that over distances and times sufficiently large to affect the relative motions of interacting cloud droplets the turbulence can be regarded as a linear shear. They give evidence that the presence of a shear increases the collection efficiencies over those in still air. The techniques used by these workers

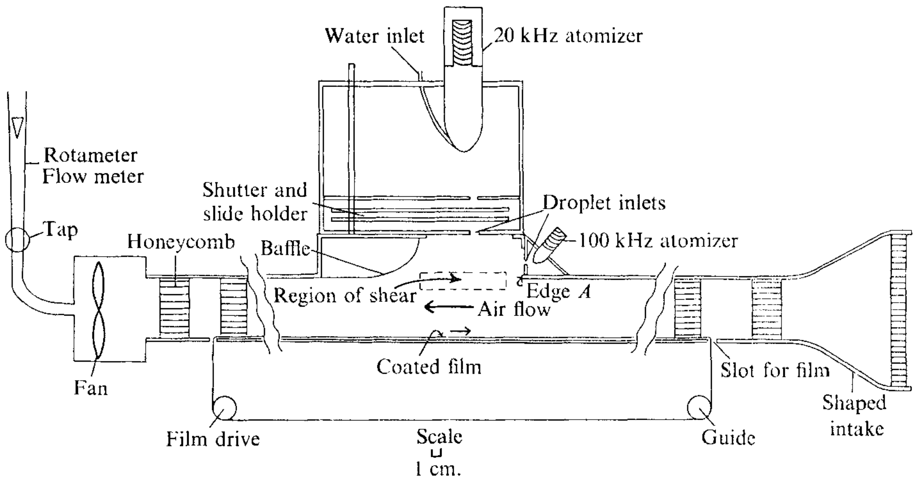

FIGURE 1. Diagram of the apparatus for measuring collection efficiencies in a sheared flow.

have been modified to obtain more reliable results and these are described in this paper. The range of the experimental results has been extended to include more of the conditions expected in developing turbulent clouds.

2. Experimental apparatus and techniques

The apparatus, shown in figure 1, was a modification of the wind tunnel described by Woods *et al.* (1972). The wind tunnel was about 1 m long and of rectangular cross-section, being 60 mm by 100 mm except for the 100 mm square central section. The air flow was straightened by the honeycomb sections and an estimate of the mean flow through the tunnel could be obtained from the rotameter. Two transverse slots cut in the floor of the tunnel enabled a length of specially coated 35 mm film to be moved along the floor by an electric motor.

Droplets were introduced into the tunnel through slots cut in its roof and in the vertical face above the edge A. The positions into which they fell were altered by sliding the entry positions along the slots. Droplets produced by 100 kHz ultrasonic atomizer (Brownscombe 1966) were used as tracers to investigate the flow in the tunnel by the 'streak photography' method described by Woods *et al.* The air flow was found to be essentially straight, for mean speeds of up to about 150 mm s^{-1}, with a region of uniform shear between the stagnant air in the expanded region downstream from A and the uniform flow in the tunnel. A typical velocity map, constructed from a large number of streak photographs, is shown in figure 2 (a). A vertical velocity profile is shown in figure 2 (b).

The experiment was designed to detect coalescence between individual droplets produced by one atomizer and those of a different size produced by another. One atomizer produced an almost monodispersed cloud of droplets doped with sodium chloride while the other produced larger droplets of distilled water. Droplets containing sodium chloride landing on the floor of the tunnel were

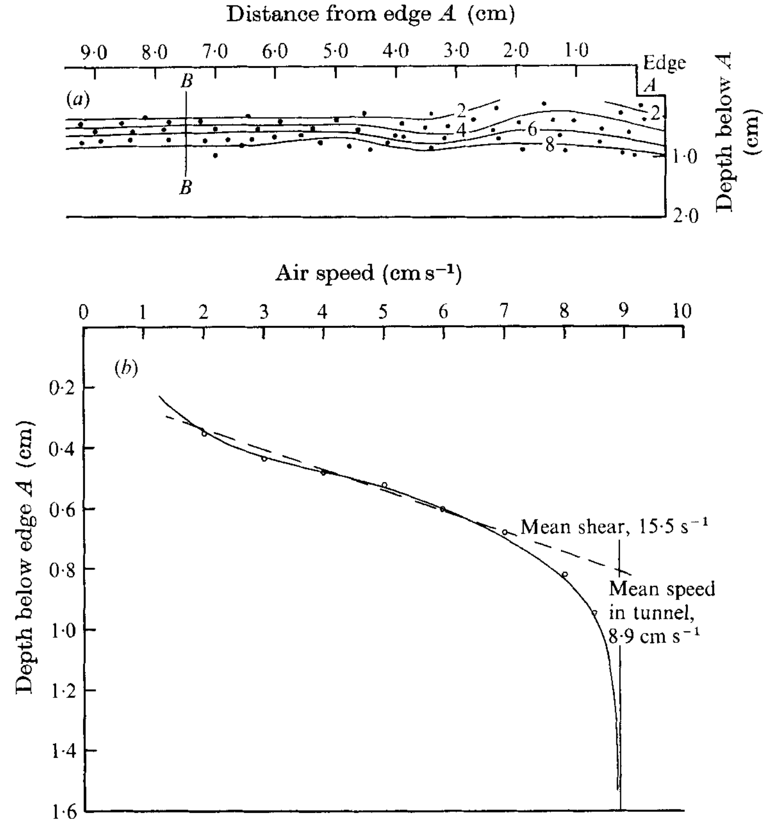

FIGURE 2. (*a*) Velocity map of the shear with isotachs at 2 cm s^{-1} intervals constructed from a series of streak photographs. The dots indicate the points at which the streaks were measured. (*b*) Velocity profile through section *B*–*B*.

therefore either the smaller droplets or droplets formed by the coalescence of one of these droplets with another droplet.

The film was coated with a freshly prepared solution of 5 g gelatine, 2 g silver nitrate and 6 drops of nitric acid in 100 ml of warm distilled water and allowed to dry in a cool darkened room. A droplet of radius less than about 60 μm landing on this film at terminal velocity would leave no impression if it contained no sodium chloride. Doped droplets, however, left marks which turned into dark spots after exposure to ultra-violet light. The radii of these spots were insensitive to the concentration of sodium chloride in the droplets but were 2·4 times as large as the droplets causing them. Using such a film, doped droplets landing on the floor of the tunnel could be detected, and from measurements of the radii of the spots it was possible to determine whether these droplets were the original small droplets or were the result of coalescence with a larger droplet.

The number densities of droplets moving in a cloud in the wind tunnel were determined either by counting the number of streaks in focus on streak photographs of the cloud or by collecting the droplets under oil for a known time and weighing them. These methods agreed to within $\pm 10\,\%$ for clouds of the densities used in the experiments. In general the photographic method was preferred because it could be used without disturbing the air flow in the wind tunnel and also because the use of a stroboscopic lamp enabled the air velocity in the region occupied by the droplets to be measured.

The size distributions of droplets in the clouds were determined by allowing them to settle onto magnesium oxide coated glass slides and measuring the craters which were made in the coating (May 1950). An alternative method, used for the sodium chloride doped droplets, was to allow them to settle onto the coated film and measuring the dark spots. These methods gave agreement to within $\pm 3\%$ for droplets of radii within the range of interest, falling at their terminal velocities.

In the first experiments, illustrated in figure 1, droplets were produced using two ultrasonic atomizers. A 20 kHz atomizer producing droplets with a wide spread of sizes was mounted above the tunnel so that the droplets fell through a hole in the roof. An almost monodispersed cloud of droplets was obtained by passing droplets from a 100 kHz atomizer through a system of baffles. This cloud was introduced into the tunnel through a slit in the vertical wall above the edge A. The droplet spectrum produced by each was constant provided that the water flow rate was constant; typical spectra are shown in figure 3. The droplets from the 100 kHz atomizer constituted the smaller droplets in the experiment and these were doped with sodium chloride.

The shear was set by adjusting the mean flow through the tunnel. After the film had been started in motion along the tunnel floor at about $5\,\mathrm{mm\,s^{-1}}$ the atomizers were turned on. Streak photographs of the interacting droplets were taken throughout the experiment using both stroboscopic and continuous illumination. Droplet spectra were taken from both atomizers before and after the experiment, which was continued until the coated films had passed through the tunnel. The duration of the experiment was usually of the order of 60 s and this was noted in order that the film speed through the tunnel could be calculated.

After such an experimental run the coated film was exposed to ultra-violet light and the dark spots on it were measured using a microscope with a graduated eyepiece. Usually several hundred impressions, which were confined to the centre of the film, were measured at various points along the film. The movement of the film not only avoided variations of their size spectra along the film due to the differing trajectories of different sized droplets but also prevented overlapping impressions. Any experiments showing variations were rejected as this implied changes in the droplet input spectra. The total number of droplets landing was calculated from the number of impressions on measured areas of the film. The input rate of the larger droplets was calculated from the streak photographs. This information, together with the size spectrum obtained from the magnesium oxide slides enabled the number n_i of larger droplets injected into the tunnel to be calculated for a series of size classifications from $10\,\mu$m to about $32\,\mu$m. The spectrum of the smaller droplets (figure 3(*b*)) was very narrow and in this experiment all droplets were assumed to be of radius $9\,\mu$m. From the spectrum and total number of the salt droplet impressions, the numbers of combined droplets n_0 due to each of the size classifications were obtained.

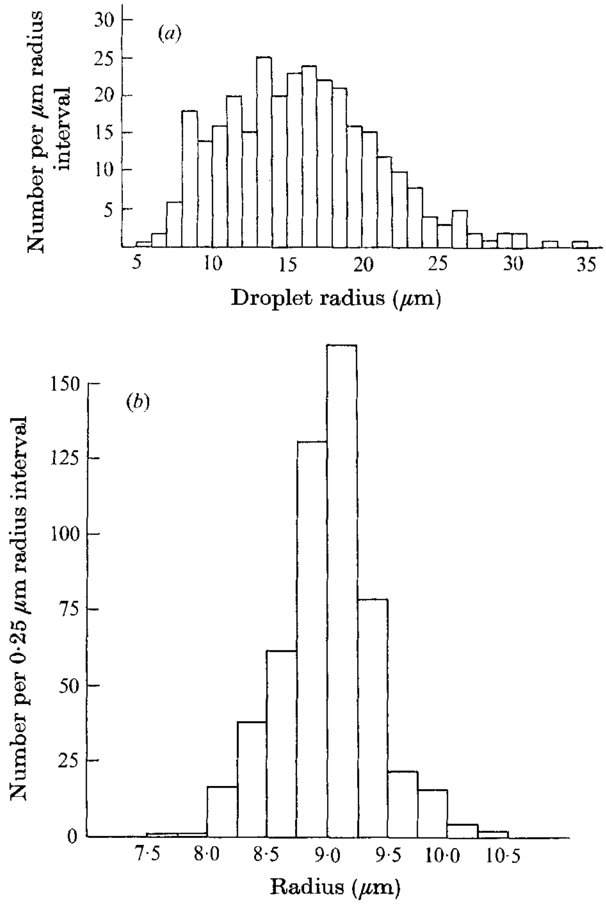

FIGURE 3. Droplet spectrum from (*a*) the 20 kHz ultrasonic atomizer and (*b*) the 100 kHz atomizer after the cloud had passed through a system of baffles.

The collection efficiency is given by

$$E(R, r) = n_0/(n_i n d\pi(R+r)^2),$$

where R is the radius of the larger droplet and r that of the smaller; n and d (obtained from streak photographs) are the number density and depth of the cloud of smaller droplets respectively. Even in the most extreme cases used in these experiments the fact that the relative trajectories of the droplets are not strictly vertical can be neglected in calculating the results. To extend the range of the larger droplets studied, the 20 kHz atomizer was replaced in later experiments by the vibrating-needle atomizer (Mason, Jayaratne & Woods 1963) adjusted to produce droplets up to 40 μm radius.

The experiments were repeated several times using different droplet cloud densities and for shears ranging from 8 s^{-1} to 18 s^{-1}. Lower shears could not be used as these were produced only with low air speeds in the tunnel at which smaller droplets would have fallen through the shear region before interaction could have occurred. At shears higher than $18\,s^{-1}$ the air flow in the tunnel became unstable.

Some of the experiments were repeated with the 100 kHz atomizer moved further upstream so that the clouds of droplets were observed to interact in a region lower in the shear. Later, the atomizer was moved still further upstream so that the interactions were in the region of the tunnel where the velocity was constant. This should have been equivalent to running the experiment in still air, assuming that the droplets had all acquired the horizontal air velocity, as a constant velocity superimposed on both droplets should not affect their relative motion. Measurements obtained from the streak photographs showed that the droplets had all acquired the same horizontal velocity.

In order to extend the range of sizes of the smaller droplets an air driven spinning-top atomizer (May 1949) was used for these droplets. It was found that a monodispersed cloud of droplets with radii in the range 10–50 μm could be produced. However, the rapidly rotating top induced very violent air motion in the enclosed space containing it, and it was difficult to prevent this from interfering with the laminar air flow in the tunnel, especially when the flow was slow. Nevertheless, it was possible to introduce droplets from the clouds produced by the spinning top into the linear shear if the velocity in the tunnel was above about $100\,mm\,s^{-1}$, but this restricted the results to a shear of $14 \pm 1\,s^{-1}$. For this shear a wide range of interacting droplet radii was studied. Droplets smaller than 9 μm radius were not used because evaporation of the droplets increases rapidly as the radius is decreased below this figure.

Some experiments were also made with the wind tunnel inclined at an angle to the horizontal, again using the two ultrasonic atomizers to produce the droplets; further experiments were made with the mean relative humidity in the wind tunnel increased from about 70 % to over 96 %.

3. Results

The results of the first experiments are shown in figure 4, where the collection efficiency of larger droplets for 9 μm radius droplets is shown for several shears, the points being the mean results from about 3 or 4 experimental runs using different cloud densities. There was no systematic difference between different runs, indicating that the collection efficiencies were independent of the cloud density. The theoretical results of Hocking & Jonas (1970) for the same droplets falling in still air are shown for comparison. The collection efficiencies of droplets of radius less than about 25 μm were much larger in the shear than those that still-air theory would predict but those for larger droplets were very little affected by the shear. The increase in the collection efficiencies of the smaller droplets was greater for higher shears.

Figure 5 shows the results obtained when the atomizer was moved upstream. The still-air theoretical results and some of the results of Woods *et al.* (1972) are also shown. The results of Woods *et al.* are seen to be in good agreement with the present work. The collection efficiencies obtained with the interactions occurring below the shear are in very good agreement with the still-air theory,

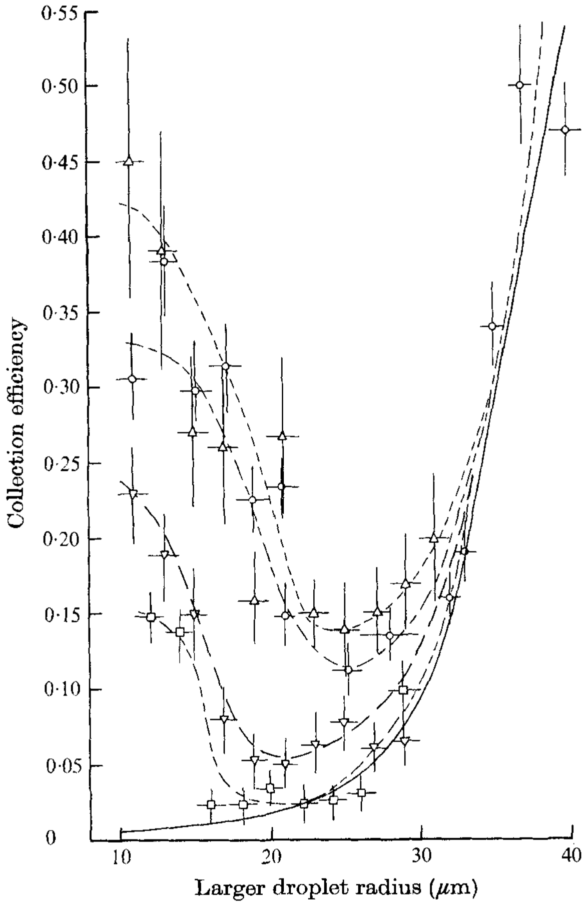

FIGURE 4. Experimentally determined collection efficiencies of larger droplets for 9 μm radius droplets falling through a linear shear in the horizontal wind. ---△---, shear = 18 s^{-1}; ---○---, shear = 14 s^{-1}; ——▽——, shear = 10 s^{-1}; ——□——, shear = 8 s^{-1}; ———, theoretical curve for droplets falling in still air (after Hocking & Jonas 1970).

thus justifying the experimental technique, while the other results indicate that the position of the interaction in the shear is not important. The results obtained with a high relative humidity were not significantly different from those obtained with lower humidities.

The spinning-top atomizer produced a wide range of the smaller droplet sizes and the results obtained using these are shown in figure 6, where the collection efficiencies for several larger droplet radii are plotted as a function of the ratio of the interacting droplet radii. All of these results are for a shear of 14 s^{-1}. Comparison with the still-air theory shows that even with a wide range of smaller droplet sizes the collection efficiencies of droplets greater than 25 μm radius are very little affected by the shear.

The effects of tilting the wind tunnel are shown in figure 7, where it is seen that effects of the shear are reduced as the inclination of the tunnel is increased, implying that the orientation of the shear is important in the determination of the

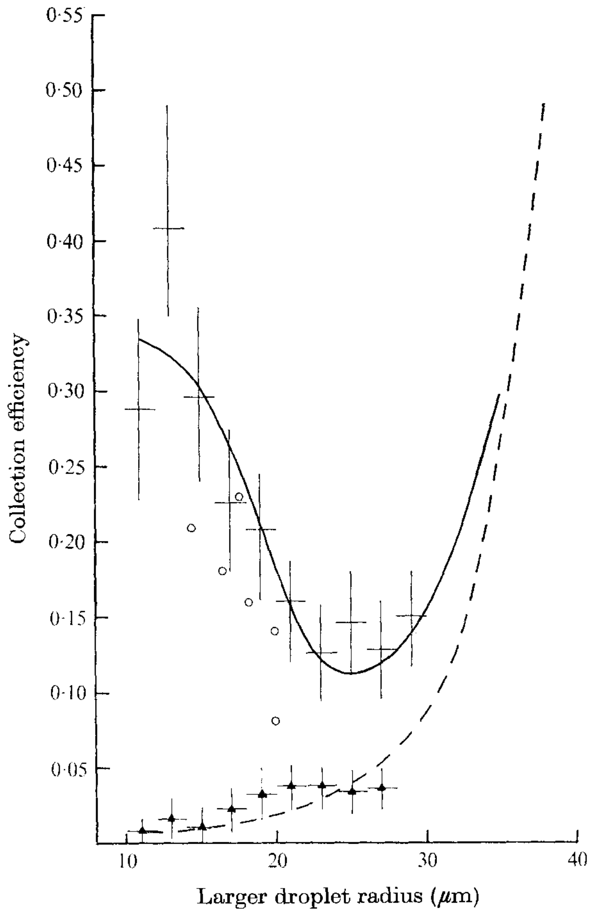

FIGURE 5. Diagram showing the variation with the position of the interaction in the shear of the collection efficiency for 9 μm radius droplets and the results of Woods *et al.* (1972). †, values obtained using the lower part of a shear of $14\,s^{-1}$; ▲, values obtained with the interaction below the shear; ———, results obtained using the entire depth of the shear of $14\,s^{-1}$; — — —, theoretical curve for still air; O, results of Woods *et al.* for a shear of $17{\cdot}5\,s^{-1}$.

collection efficiency. $\delta E(R, r, S)$, the difference between the measured collection efficiencies in a shear of magnitude S and the theoretical values for the same droplets in still air, can be considered as a measure of the effects of the shear. In figure 8 this quantity is plotted as a function of S for several values of R with $r = 9\,\mu m$. These results suggest that we can write

$$\delta E(R, r, S) = k(R, r) \times [S - S_0(R, r)],$$

where $S_0(R, r)$ represents some threshold shear flow below which the shear has no effect and $k(R, r)$ represents the variation of the effects of the shear with the droplet radii.

Before considering the explanation of the very large effect of the shear on the collection efficiency it is necessary to consider the possible sources of error in the experiment and how these could affect the reliability of the results. The main

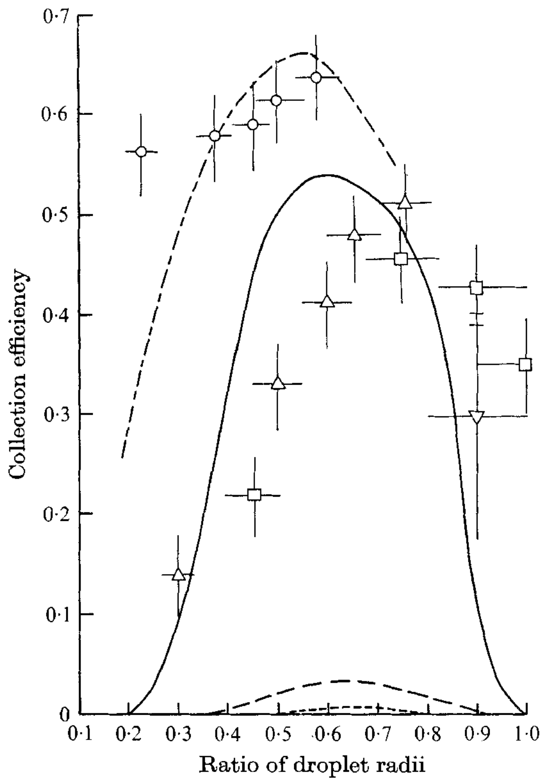

FIGURE 6. Collection efficiencies in a shear of $14\,s^{-1}$ in a horizontal wind. The lines represent the theoretical results for still air and the points the experimental results. The results are shown for several larger droplet radii; ○, — — — —, 40 μm; △, ————, 30 μm; □, — —, 20 μm; ▽ - - - -, 10 μm.

sources of error in the experiments were believed to be the errors in the determination of the cloud densities. There were systematic errors of about 10 % in each experiment but averaging results obtained using several different cloud densities reduced this error to about 5 %. The errors introduced by the sampling of the droplet clouds were limited to about 3 %, which was comparable with the reproducibility of the spectra, by the appropriate choice of the class widths and the sizes of the samples.

The results have been calculated assuming that only one droplet would be captured by a larger droplet. Multiple coalescence could affect the results but the probability of this was only about 0·25 % since, on average, only about 5 % of the larger droplets underwent a single coalescence. The collection efficiencies of almost equally sized droplets in the shear have been shown to be very large and it might be thought that some modification of the smaller droplet spectrum would occur between the sampling and interaction areas. However, even for droplets at opposite ends of the spectrum only an insignificant proportion of the droplets could coalesce in the time available and samples taken in separate runs at the shear region showed no detectable difference from those obtained in the normal position.

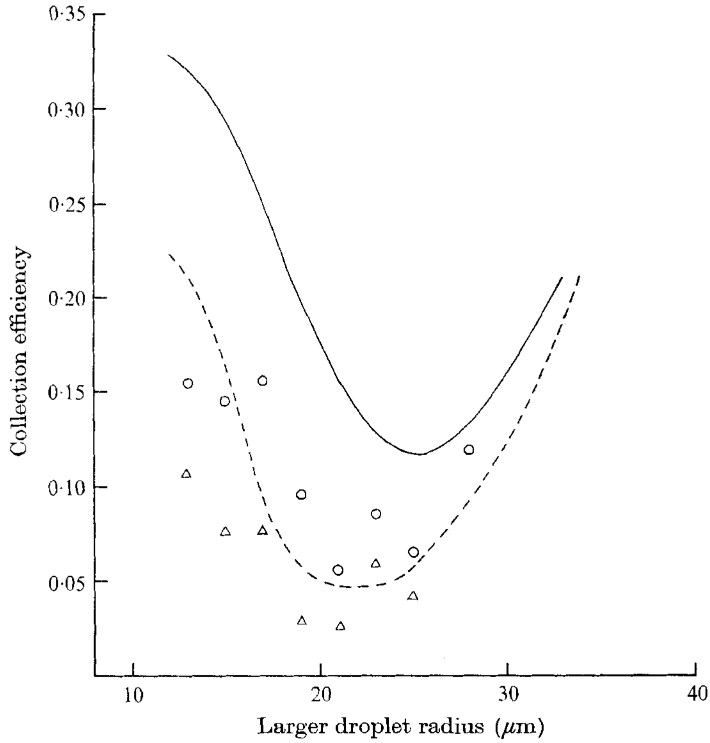

FIGURE 7. The effects of the inclination of the wind on the experimental collection efficiencies for 9 μm radius droplets. ——, results obtained in a shear of $14\,s^{-1}$ in a horizontal wind tunnel; – – –, similar results in a shear of $10\,s^{-1}$. Results with the tunnel inclined at 45° to the horizontal: ○, shear of $14\,s^{-1}$; △, shear of $10\,s^{-1}$.

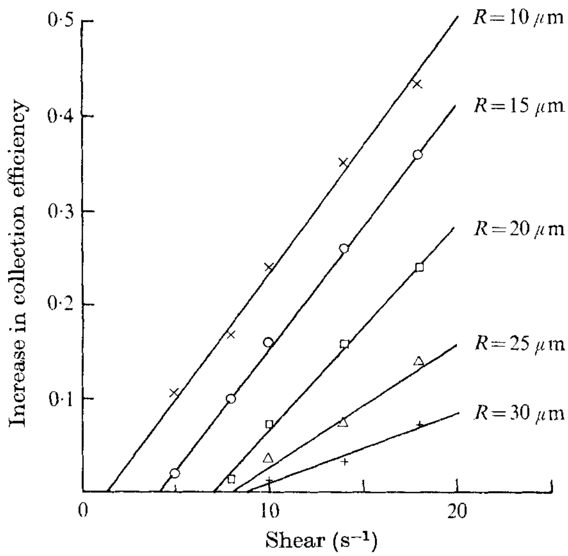

FIGURE 8. Diagram showing the increase in the collection efficiencies, as a function of the shear, for 9 μm radius droplets caused by the shear for several radii of the larger droplets.

The possible entrainment of the smaller droplets into the cloud of larger droplets without coalescence could have given rise to spuriously large collection efficiencies. In these experiments entrainment was avoided by making the cloud densities as low as was consistent with obtaining a reasonable number of coalescences. That entrainment had not occurred was confirmed because drops brought down by entrainment would have given smaller impressions on the film than droplets resulting from coalescence with a larger droplet. Separate tests showed that if a droplet containing sodium chloride landed on top of a pure water drop a distinctive impression was made.

The effects of any electric charges on the drops are believed to be very small. The charges on droplets produced by disrupting films of water have been measured by Jonas & Mason (1968) and are typically of the order of 10^{-15} C. The calculations of Davis (1964) show that the forces on colliding cloud droplets due to charges of this magnitude are much smaller than the hydrodynamic forces, so that these charges would have a negligible effect on the collection efficiencies. The agreement between the theoretical results in still air and the experimental results at constant velocity also suggests that the effects of electric charges are small.

It is thought that these experimental results are reliable measurements of the collection efficiencies of cloud droplets falling through a linear sheared air flow and that they are accurate to within $\pm 10\%$.

4. Theory

Very little previous work has been reported on the interactions of bodies of comparable sizes in a sheared fluid. Lin, Lee & Sather (1970) have considered the motion of neutrally buoyant spheres in a sheared fluid but the assumption of neutral buoyancy makes the results inapplicable to the present problem.

The radii of the droplets considered in these calculations were less than $30\,\mu$m and their velocities were comparable with their terminal velocities in still air. The Reynolds number of the flow around the droplets was therefore less than unity, as was that of a shear flow of magnitude $20\,s^{-1}$ extending over a few millimetres. Under these conditions the flow around the droplets, regarded as rigid spheres, can be calculated using Stokes's equations. Because of the linearity of these equations the forces on spheres moving in a sheared air flow can be separated into components arising from the shear and from the motion of the spheres. Davis (1971) calculated and tabulated the forces on a pair of stationary spheres in a sheared flow in terms of coefficients which depend on the ratio of the sphere radii and their separation, using a similar technique to that used by Lin *et al.* The forces due to the movement of the spheres have been calculated by Jeffery (1915), Stimson & Jeffery (1926) and Davis (1969).

The trajectories of spheres in the sheared air flow can be found by solving the equations of motion of spheres under the action of aerodynamic and gravitational forces. Trajectories giving collisions between the spheres should then be obtainable. As Hocking & Jonas (1970) pointed out, however, the forces of repulsion between the spheres due to their movement, calculated assuming that for any gap between them the air is a continuous fluid, increase indefinitely as the gap is decreased. This would prevent any collisions, but the assumption of a continuous fluid clearly breaks down when the gap between the spheres is less than the mean free path of the gas molecules. To avoid this difficulty Hocking & Jonas used as a criterion for a collision the fact that the gap between the spheres became less than some fraction ϵ of the larger droplet radius. They used a value of ϵ of 10^{-4} but recent calculations by Hocking (1972) indicate that a value of ϵ of 10^{-2} or 10^{-3} would probably be more accurate. In this paper a 'collision' will mean that the gap had decreased to less than 10^{-3} of the larger sphere radius.

Large drop radius (μm)	Ratio of drop radii	Calculated collision efficiency: Still air	Calculated collision efficiency: Shear $20\,s^{-1}$	Experimental collection efficiency Shear $14\,s^{-1}$
10·0	0·2	0·005	0·005	—
10·0	0·5	0·013	0·013	—
10·0	0·9	0·018	0·019	0·30
20·0	0·2	0·007	0·007	—
20·0	0·5	0·04	0·04	0·27
20·0	0·5	0·04	0·07†	—
20·0	0·9	0·021	0·025	0·43
20·0	0·99	0·016	0·06	0·33
30·0	0·2	0·03	0·03	—
30·0	0·5	0·5	0·5	0·33
30·0	0·9	0·11	0·13	—
30·0	0·99	0·025	0·06	—

TABLE 1. Comparison of the experimental collection efficiencies of cloud droplets in still air and the collision efficiencies in a vertically sheared horizontal air flow, calculated assuming $\epsilon = 10^{-3}$

† Calculated for a shear of $50\,s^{-1}$.

The collision efficiency between two spheres can in general be calculated from the cross-sectional area of the volume enclosed by grazing trajectories of the smaller sphere relative to the larger one, and in this paper will be defined by

$$E = A/\pi(a_1 + a_2)^2,$$

where A is the horizontal cross-sectional area and a_1 and a_2 are the sphere radii. In the case of collisions in still air the volume enclosed is a vertical cylinder but this is not, in general, true for collisions in moving air. Details of the calculations of individual relative trajectories of the spheres under the action of the aerodynamic and gravitational forces are given in an appendix to this paper. Using these trajectories, the collision efficiencies of pairs of spheres moving in a sheared air flow of arbitrary orientation were calculated using the definition of collision efficiency given above. The collision efficiencies for three values of the radii of the larger sphere and for several values of the radius ratio were calculated for a shear of $20\,s^{-1}$ in a horizontal air flow. These are shown in table 1, where they are compared with the experimental observations.

Calculations were also made of the collision efficiencies of the droplets in shears consisting of a horizontal velocity gradient in both horizontal and vertical air flows. There was no significant change in the collision efficiencies compared with those in still air for any of the pairs of droplet radii used. The effect of the shear was to alter the shape of the cross-section of the volume swept out by the larger droplet without changing its cross-sectional area.

5. Discussion and conclusions

The results of the experiments described in this paper indicate that the collection efficiencies of small droplets falling in still air are appreciably increased when the droplets fall through a linear shear of between 8 and 18 s^{-1}. Calculations of the relative trajectories of pairs of interacting droplets in a shear do not explain this increase.

The major assumptions in the calculations of the relative trajectories, that the air flow could be represented by the Stokes equations and that the droplets could be regarded as rigid spheres, have been considered by Hocking & Jonas (1970), who showed that errors introduced by them would be small. Also, errors introduced by these assumptions are greater for larger droplets, whereas the discrepancy between the present calculations and the experiments is large for smaller droplets. In the calculation of the collision efficiencies from the relative trajectories it was assumed that the same criterion could be used to define a collision as was used in the still-air calculations. Although the use of a value of ϵ of 10^{-3} in the calculations of the collision efficiencies in still air gives results which are in reasonable agreement with the present experimental results in air moving with constant velocity, it is pertinent to examine whether the same value should be used in the calculations for droplets in a shear flow.

The streak photographs which were used to examine the air flow in the wind tunnel showed that there was no turbulence affecting the motion of the droplets on scales of the order of tens of microns. They did not however rule out the possibility of motion on smaller scales. It is possible that collisions were promoted by small-scale air motions between droplets which were close together but which would not be expected to collide from relative trajectory considerations. The effect of this small-scale motion would be to increase the value of ϵ which should have been used in the collision calculations. When calculations of the collision efficiencies were carried out using increased values of ϵ it was found that the experimental results could be explained with a value of 0·1 for the smallest droplets considered, decreasing to 10^{-3} for droplets of radius greater than about 25 μm. It is possible, therefore, that the effect of the shear is to alter the behaviour of the droplets when they are close together, perhaps through the mechanism of small-scale air motion, thereby increasing the collection efficiencies. It should be pointed out, however, that the experimental result showing a decreased effect in a tilted shear is difficult to reconcile with this explanation.

The conclusions reached in this paper suggest several regions where further research effort will be needed. At present, while a quantitative explanation of the discrepancy between the experimental and theoretical results cannot be given, the results should be applied with caution. However, it is seen that there is more scope for improvement of the calculations than in the experimental results and, therefore, that the latter should be more reliable for application.

The authors are grateful to Dr M. H. Davis and Dr L. M. Hocking for their discussions on this problem. Many of the calculations described in this paper were completed while one of the authors (P. R. J.) was a visitor to the National Center for Atmospheric Research, Boulder, Colorado and the use of the computing facility is gratefully acknowledged. This paper is published with the permission of the Director-General of the Meteorological Office.

Appendix. The calculation of the relative trajectories of spheres in a sheared air flow

Consider the motion of two spheres of radii a_1 and a_2 ($a_1 < a_2$) in a sheared air flow. Let the magnitude of the shear be S and let it be orientated with the plane of zero velocity, inclined at some angle β to the horizontal, with the air velocity at an angle α to the line of greatest slope in that plane ($\alpha = 0$ if $\beta = 0$). The forces and couples which act on the spheres, assuming that the Stokes flow approximation to the air motion is valid, have been given by Jeffery (1915), Stimson & Jeffery (1926), Davis (1969) and Davis (1971). From these formulations the forces and couples acting on the spheres owing to their motion and owing to the shear can be calculated. These can be combined because of the linearity of the Stokes equations, and the equations of motion of the larger sphere can be written in the form

$$\begin{aligned}
du_{1x}/dt &= k_1\{C_1 u_{1x} + C_2 u_{2x} + S(C_1+C_2)\, h\cos\theta\cos\chi \\
&\qquad + a_1[C_3 w_{1y} + C_4 w_{2y} + S\cos\theta\,(C_5\cos^2\chi + C_6\sin^2\chi)]\},\\
du_{1y}/dt &= k_1\{C_1 u_{1y} + C_2 u_{2y} + S(C_1+C_2)\, h\sin\theta \\
&\qquad + a_1[-C_3 w_{1x} - C_4 w_{2x} + S\sin\theta(C_5\cos\chi)]\},\\
du_{1z}/dt &= k_1\{C_7 u_{1z} + C_8 u_{2z} + S(C_7+C_8)\, h\cos\theta\sin\chi \\
&\qquad + a_1 S\cos\theta(C_9\cos\chi\sin\chi)\},\\
dw_{1x}/dt &= k_2\{C_{10} u_{1y} + C_{11} u_{2y} - S(C_{10}+C_{11})\, h\sin\theta \\
&\qquad + a_1[C_{12} w_{1x} + C_{13} W_{2x} + S\sin\theta(C_{14}\cos\chi)]\},\\
dw_{1y}/dt &= k_2\{-C_{10} u_{1x} - C_{11} u_{2x} - S(C_{10}+C_{11})\, h\cos\theta\cos\chi \\
&\qquad + a_1[C_{12} w_{1y} + C_{13} w_{2y} + S\cos\theta(C_{14}\cos^2\chi + C_{15}\sin^2\chi)]\},\\
dw_{1z}/dt &= k_2 a_1[C_{16} w_{1z} + C_{17} w_{2z} + S\sin\theta(C_{18}\sin\chi)],
\end{aligned}$$

where, if ν and ρ are the viscosity and density of water respectively, $k_1 = 3\nu/a_1^2\rho$, $k_2 = 2{\cdot}5k_1/a_1$. The constants C_1–C_{18} are functions of the droplet separation and ratio of their radii and can be obtained from the results cited earlier. In these equations the term u_{1x} represents the velocity of droplet 1 (the larger) parallel to the x axis, u_{1y} its velocity parallel to the y axis, etc., while w_{1x}, w_{1y}, etc. are its angular velocities about those axes. These axes are defined with the z axis along the line of centres of the spheres, directed from the larger, and the x axis is parallel to the plane of zero air velocity. The complement of the angle between the air velocity and the y axis is θ and χ is the angle between the z axis and the direction of the velocity gradient in the air. The distance of the origin of this co-ordinate system from the plane of zero velocity is denoted by h; this origin divides the line between the sphere centres (of length c) in the ratio $(c^2+a_1^2-a_2^2):(c^2-a_1^2+a_2^2)$.

A similar set of equations, with different constants, can be obtained for the smaller sphere and the resulting set of twelve equations can be used to calculate the linear and angular accelerations of the droplets under the action of the aerodynamic forces.

The relative trajectories of the spheres, which in general do not remain in one plane, under the action of both aerodynamic and gravitational forces can be obtained by integration of the equations of motion of the spheres in some fixed co-ordinate system. The changes in the velocities and angular velocities of the spheres can be calculated by resolving the equations given above into the fixed system of co-ordinates and then the gravitational effects can be added. The changes in the absolute and relative co-ordinates of the centres of the spheres can then be calculated. The relative trajectories are obtained by successive application of this integration procedure.

The integrations were carried out until the smaller sphere, initially some distance below the larger one, had passed and was above the larger sphere. The initial position of the larger sphere was chosen as the origin of the fixed coordinate system. The smaller sphere was initially at some point in a horizontal plane at a distance of at least ten large-sphere radii below the larger sphere, where the aerodynamic forces on it due to the larger sphere could be safely assumed to be negligible. Tests showed that when this factor was increased to twenty-five there was no change in the relative trajectories. The choice of the initial position of the larger sphere on the plane of zero velocity did not introduce any loss of generality as a displacement would have been equivalent to imposing a constant velocity on the system. This would not have affected the relative motion. The initial linear and angular velocities assigned to the spheres were those which the droplets would have had if they had reached the initial positions for these calculations after a long fall through the shear in the absence of the other sphere.

The numerical integrations were carried out using a technique which enabled the step length to be altered to maintain the accuracy of the calculations. A tolerance of 0·1 % was usually allowed as this resulted in trajectories which were not significantly different from those obtained using a much smaller tolerance. The step length was usually much less than 10^{-4} s.

REFERENCES

Bartlett, J. T. 1970 The effect of revised collision efficiencies on the growth of cloud droplets by coalescence. *Quart. J. Roy. Met. Soc.* **96**, 730–738.

Bartlett, J. T. & Jonas, P. R. 1972 On the dispersion of the sizes of droplets growing by condensation in turbulent clouds. *Quart. J. Roy. Met. Soc.* **98**, 150–164.

Brownscombe, J. L. 1966 The electrification of ice and the charging of rime deposits. Ph.D. thesis, University of London.

Davis, M. H. 1964 Two charged spherical conductors in a uniform electric field: forces and field strength. *Quart. J. Mech. Appl. Math.* **17**, 499–511.

Davis, M. H. 1969 The slow translation and rotation of two unequal spheres in a viscous fluid. *Chem. Engng. Sci.* **24**, 1769–1776.

Davis, M. H. 1971 Two unequal spheres in a slow viscous linear shear flow. *National Center for Atmospheric Research, Boulder, Colorado Tech Note*, NCAR-TN/STR-64.

Hocking, L. M. 1972 The effect of slip on the motion of a sphere close to a wall and of two adjacent spheres. Submitted to *J. Fluid Mech.*

Hocking, L. M. & Jonas, P. R. 1970 The collision efficiency of small drops. *Quart. J. Roy. Met. Soc.* **96**, 722–729.

Jeffery. G. B. 1915 On the steady rotation of a solid of revolution in a viscous fluid. *Proc. Lond. Math. Soc.* **14**, 327–338.

Jonas, P. R. & Mason, B. J. 1968 Systematic charging of water droplets produced by breakup of liquid jets and filaments. *Trans. Faraday Soc.* **64**, 1971–1982.

Lin, C. J., Lee, K. J. & Sather, N. F. 1970 Slow motion of two spheres in a shear field. *J. Fluid Mech.* **43**, 35–47.

Mason, B. J. 1971 *The Physics of Clouds*. Oxford University Press.

Mason, B. J., Jayaratne, O. W. & Woods, J. D. 1963 An improved vibrating capillary device for producing uniform water droplets of 15 to 500 μm radius. *J. Sci. Instrum.* **40**, 547–249.

May, K. R. 1949 An improved spinning top homogeneous spray apparatus. *J. Appl. Phys.* **20**, 932–938.

May, K. R. 1950 The measurement of airborne droplets by the magnesium oxide method. *J. Sci. Instrum.* **27**, 128–130.

Sartor, J. D. 1957 The mutual interaction of cloud droplets in the electrostatic field of the atmosphere. *Rand Corp. Rep.* no. 1824.

Stimson, M. & Jeffery, G. B. 1926 The motion of two spheres in a viscous fluid. *Proc. Roy. Soc.* A **111**, 110–116.

Woods, J. D. 1965 The effect of electric charges upon collisions between equal sized water drops in air. *Quart. J. Roy. Met. Soc.* **91**, 353–355.

Woods, J. D., Drake, J. C. & Goldsmith, P. 1972 Coalescence in a turbulent cloud. *Quart. J. Roy. Met. Soc.* **98**, 135–149.